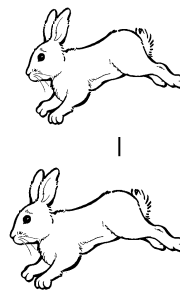


**NUMBER THEORY :
THE FIBONACCI NUMBERS**

Name: No :

Class : Date :

In A.D. 1202, **Fibonacci** posed a problem with rabbits. He asked this : if a pair of rabbits takes one month to mature, then mates, and produces a new pair the following month (1 male and 1 female) and for every month after that, **AND** their offspring do the same, how many pairs will there be after one year ?



Q.1: To work this out, can you complete the situation at End 7th i.e. T₇

<u>Month</u>	<u>Total Pair</u>
End 1 st T ₁	
End 2 nd T ₂	
End 3 rd T ₃	
End 4 th T ₄	
End 5 th T ₅	
End 6 th T ₆	
End 7 th T ₇	

So in the 1st month, pairs are immature. In the 2nd month, they become mature, mate, and by the following month produce a new pair. Mature pairs always produce a new pair the following month.

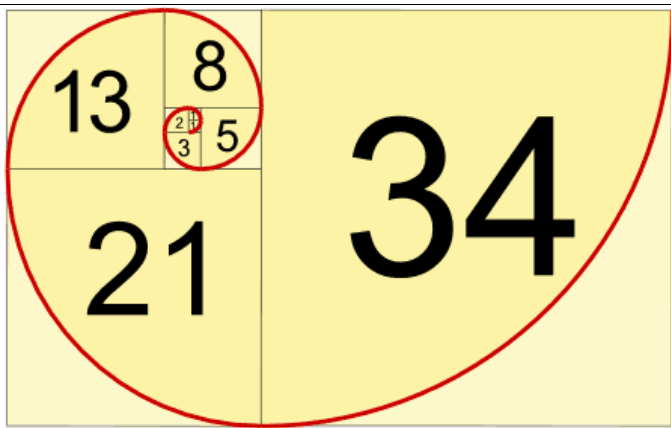
Q.2: If you can see the pattern, you can complete the table below, and answer Fibonacci's question : how many pairs of rabbits will there be after one year (at the end of the 12th Month) ? : i.e. T₁₂ =

The Fibonacci Sequence (it is infinite)

Term	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈	T ₉	T ₁₀	T ₁₁	T ₁₂	F _n =
number	1	1	2	3	5	8	?	?	?	?	?	?	F _{n.....} + F _{n.....}

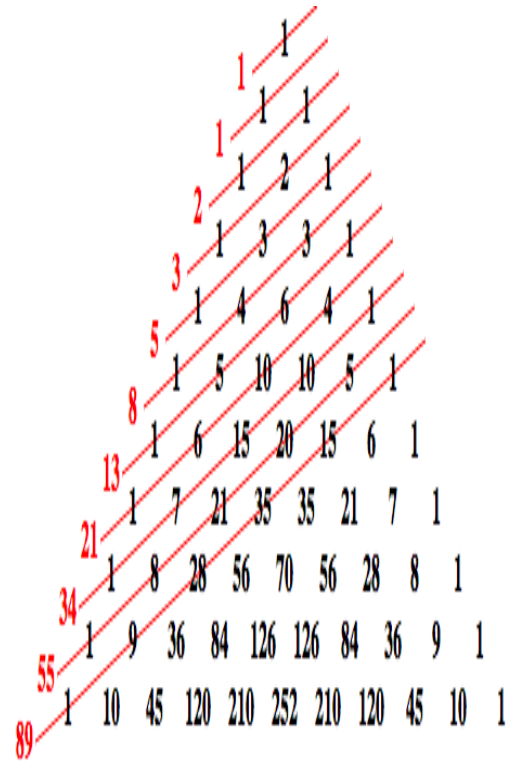
Q.4: Divide each Fibonacci number by the previous number in the sequence, and complete the table below. You can use a calculator.

$\frac{T.5}{T.4} = \frac{5}{3} = 1.66\dots$
$\frac{T.6}{T.5} = \frac{8}{5} = 1.6$
$\frac{T.7}{T.6} = \frac{13}{8} =$
$\frac{T.8}{T.7} = \frac{21}{13} =$
$\frac{T.9}{T.8} = \frac{34}{21} =$
$\frac{T.10}{T.9} = \dots =$
$\frac{T.11}{T.10} = \dots =$
$\frac{T.12}{T.11} = \dots =$
$\Phi = \frac{\sqrt{5}+1}{2}$



The quotients on the left get closer to a number known as Φ "Phi" = 1.618033 ... the "golden ratio"

Q5: Take a credit card, measure it, and divide the length by the width what is your answer? It should give you Φ . (A credit card should also fit neatly over the "golden rectangle" above.)



Above is **Pascal's triangle**. The total of the diagonal lines are the Fibonacci numbers.

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Also the sum of the squares of two consecutive Fibonacci numbers is a Fibonacci number, *e.g.*

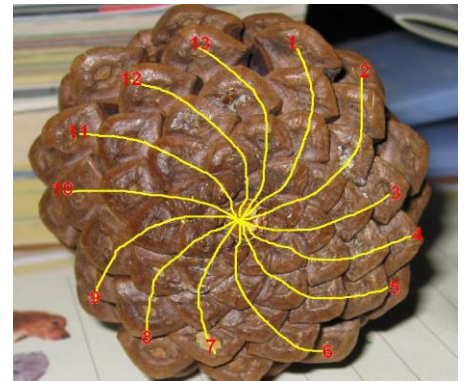
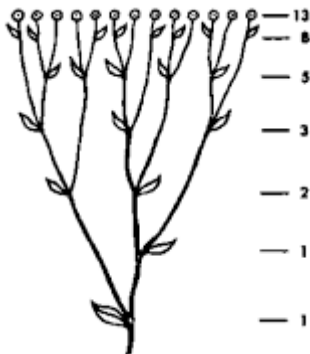
$(T_4)^2 + (T_5)^2 = 3^2 + 5^2 = 34(T_9)$

Q6: $(T_5)^2 + (T_6)^2 = \dots$

Fibonacci numbers often occur in nature, and sometimes form spirals (though there are other types of spiral in nature)

Occurring in nature, it is an optimal pattern for survival and growth.

Q7: Can you find a flower or cone yourself in nature which displays the Fibonacci sequence?



For some examples see : <http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html>