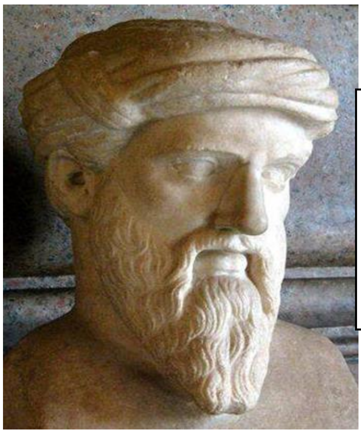
Lwr. 2nd'ry Maths Made Easy

Name : .... No : ....

Class : ....

Date : ....

Pythagoras' Theorem:

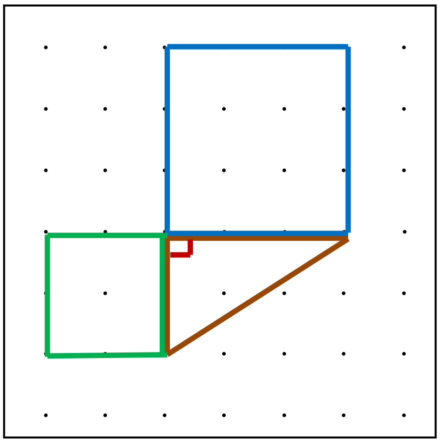


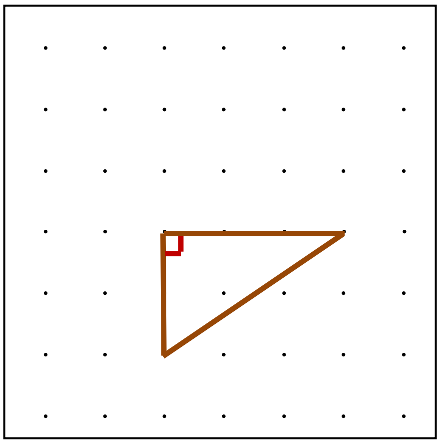
Italiano: Busto di Pitagora. Copia romana di originale greco. Musei Capitolini, Roma.Brüste des Philosophen Pythagoras von Samos in den Kapitolinischen Museen, de: Palazzo Nuovo; de: Rom Fotograf oder Zeichner: Galilea Copyright Status: de : GNU Freie Dokumentationslizenz

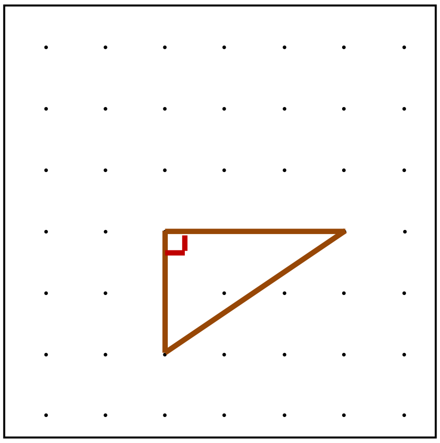
Geometric Proofs

INSTRUCTION : Do these on the worksheet, but using a 7 x 7 nail geoboard as well will make them easier to understand

Geometric Proof #1 (by shearing squares)







Make a right triangle as above and then make squares a^2 , and b^2 on the two shorter sides. Let's call the shortest side **a**

Shear both squares (keeping the same area) to the top left corner of the geoboard, forming two parallelograms.

Make a square on the hypotenuse. Shear the parallelograms to the square OR just subtract the top triangle from the shape, so $a^2 + b^2 = c^2$

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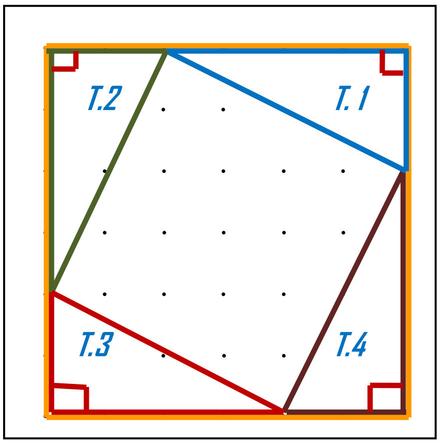
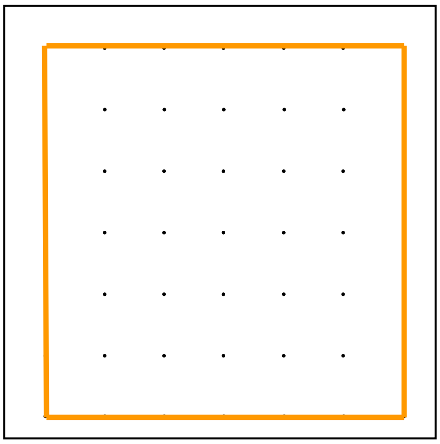
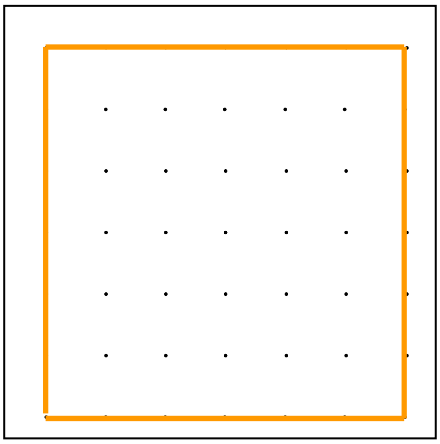
Geometric Proof #2 (by translating right triangles)





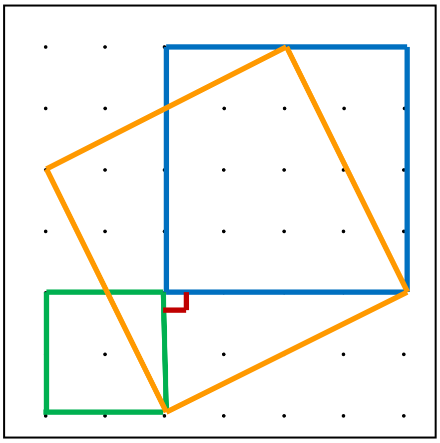


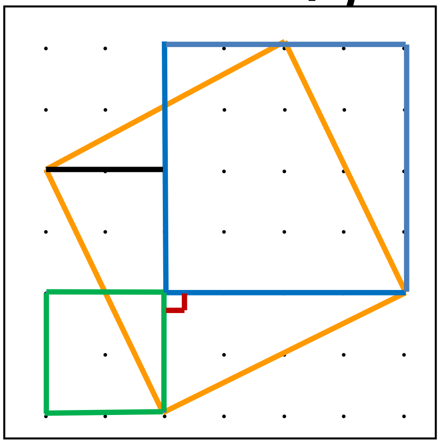
Make four separate identical right triangles as above within a large square. Note the area outside the right triangles = c^2

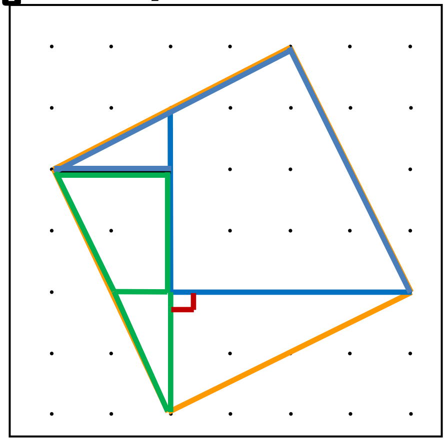
Translate Triangle 1 (T. 1) down 4 units and back 2 units so it meets T.3. The area outside the right triangles remains c^2

Translate T.4 up 2 units and T.2 across 4 units. The remaining area (outside the right triangles) remains c^2 ... so $a^2 + b^2 = c^2$

Geometric Proof #3 (by rotating areas)







Make a right triangle and then make squares a^2, b^2 and c^2 on each of the sides as above. Let's call the shortest side **a**

We are going to fill c^2. *(a)* Rotate the top right triangle of b^2 90^0 anti-clockwise.
(b) Rotate the remaining section of b^2 180^0
(c) Rotate the part of a^2 outside c^2 180^0

All sections outside c^2 have been used.
All parts of c^2 are filled
Therefore $a^2 + b^2 = c^2$

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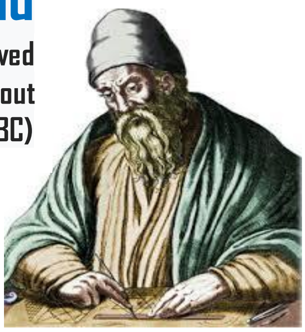
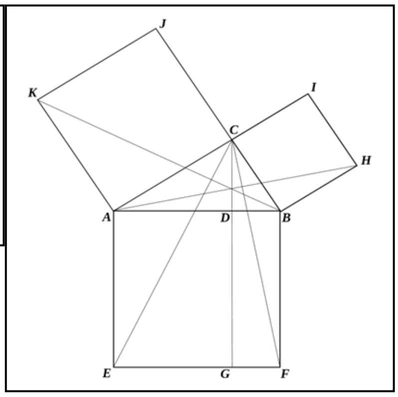
Euclid

(lived about 300 BC)

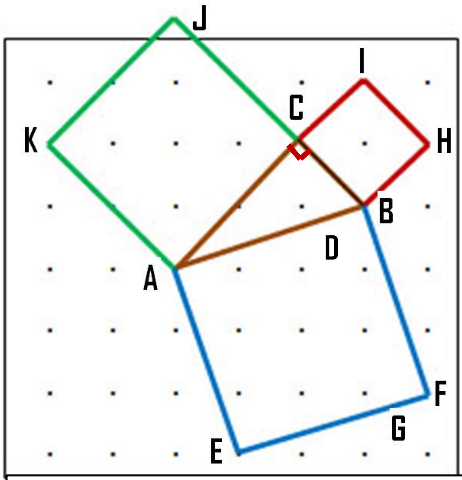


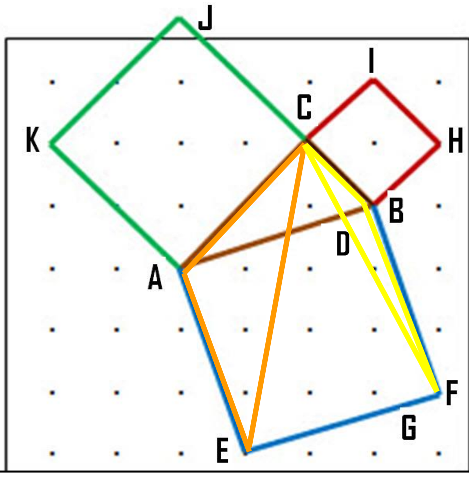
Source : Eukleides of Alexandria https://commons.wikimedia.org/wiki/File%3AEuklid-von-Alexandria_1.jpg

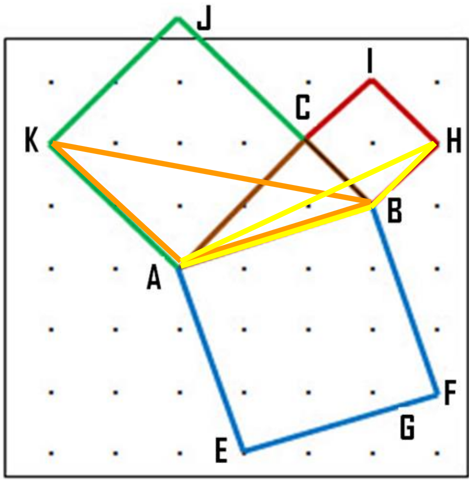
Illustration to Euclid's proof of Pythagorean theorem by congruence.by User : 4C via https://commons.wikimedia.org under *GNU Free Documentation License*.



#4. Euclid's Proof







Set up the geo-board as above, but "create" an extra nail for **"J"**

Form 2 triangles as above. By the rules of shearing, together they form **½ of c^2**

Note the larger triangle is a 90^0 anti-clockwise rotation about **A**, and the smaller triangle is a 90^0 clockwise rotation about **B**.

Euclid proves that each pair of triangles is congruent – *see at left*

Is ▲ KAB ≅ ▲ CAE ?
KA = CA and AB = AE
Angle KAB = Angle CAE (included)
∴ ▲ KAB ≅ ▲ CAE (SAS)
Note ▲ KAB = $½ b^2$

In the same way :
▲ HBA ≅ ▲ CBF (SAS)
Note ▲ HBA = $½ a^2$
So $½ a^2 + ½ b^2 = ½ c^2$
∴ $a^2 + b^2 = c^2$